A hybrid model of component sharing and platform modularity for optimal product family design

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Today’s industry faces new challenges such as diverse customer demands, shorter product development cycles and cost pressure, which compel manufacturing firms to change their production paradigm from one-size-fits-all mass production toward mass customisation. Over the past decades, modular design has received great attention as a key enabler for mass customisation, and component sharing and platform modularity have been quite popular strategies for modular design. While modular design approaches and their strategies offer a number of advantages such as late product differentiatation and changeability, there are unfortunately negative aspects, for example, sales loss due to reduced performance compared to integral design approaches, which have received little attention. Therefore, we propose a hybrid model of the two strategies in order to develop the most profitable product family. A detailed numerical analysis provides empirical support for the feasibility and effectiveness of the hybrid model.

Keywords: modular design; platform; component sharing

1. Introduction

Today’s industry faces new challenges such as diverse customer demands, shorter product development cycles and cost pressure, which compel manufacturing firms to change their production paradigm from one-size-fits-all mass production toward mass customisation. In order to meet these challenges, firms should restructure their product development processes in a more flexible manner. Many studies have proposed modular design approaches to make products at a low cost but with variety (Ulrich 1995). Since several modules (physical subsystems of a modular architecture product) are arranged to perform required functions by appropriate interfaces, (i) it is easy to change the design simply by replacing only bad or old modules for retrofit, (ii) the product development cycle can be shortened due to the fact that individual modules can be developed independently, and furthermore (iii) late product differentiation is possible, which offers a lot of benefits such as faster reaction to customer requirements and lower inventory costs. Generally, it is difficult to expect the same benefits from integral architecture products, in which required functions are implemented by only one or a few modules such that products usually have peculiar shapes and features.

Component sharing and platform modularity are quite popular strategies for modular design (Roberton and Ulrich 1998, Oshri and Newell 2005). Platforms basically are intellectual and material assets shared across a family of products (Robertson and Ulrich 1998). According to Meyer and Lehnerd (1997), a platform is a set of subsystems and interfaces developed to form a common structure from which a stream of derivative products can be efficiently developed and produced. Accordingly, a platform is the same basic structure of a product family, and other components are then assembled to it to develop individual products. Some of these components can further be shared. This is termed as component sharing. Moreover, a platform is shared across the whole product family whereas components can be shared between a subset of this family and not every product. A platform, once designed, can be deployed throughout the product family and hence significantly reduces the fixed costs of developing product variants. Another benefit of platform-based product development is its greater degree of reuse. It inspires the firms to put more effort into their design and development resulting in better architecture, tighter

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integration of components and lower unit variable costs. Once the platform is built, product variants can be developed quickly, thereby increasing the responsiveness of the firms leading to speedy delivery.

Component-sharing modularity facilitates cost-effective development and the production of a high variety of products, since the same components are used across multiple products, which allows the components to be easily removed, upgraded or replaced for redesign. Component sharing is not a new practice and has been widely used in industries. For instance, Dell uses a small set of core components but still produces various end products (Dell Computer Corporation 1994). Brake sharing among automobile families is yet another common component-sharing practice (Fisher et al. 1999). Note that the sharing of many components decreases cost but increases design constraints such as complex interface definition (Oshri and Newell 2005).

Many studies have dealt with component sharing and platform modularity independently, but hybrid approaches combining both have not yet been studied sufficiently. Furthermore, the effectiveness of hybrid approaches to diverse customer demands has received little research attention. We therefore intend to fill this void in the literature by developing a simple but easily extensible hybrid model of component sharing and platform modularity. The research questions we address in this paper are as follows:

(1) When platform modularity and component sharing are integrated, is this strategy more profitable than the other independent approaches being used today?
(2) What should be the key parameters of platform modularity and component sharing which maximise profit without degrading product quality?
(3) What should be the values of the key parameters to develop an optimal product family?
(4) Which of the four approaches, (i) Platform modularity and component sharing integrated together to develop the two products. (ii) Only platform modularity is used with the given set of components not shared. (iii) Products are developed individually with the given set of components shared. (iv) Products are developed individually with the given set of components not shared is most profitable when the values are given a priori?

The outline of this paper is as follows: Section 2 presents a brief introduction of the past work carried out in the field and the results that are to be further exploited in this paper. A detailed formulation of the model and concepts are portrayed in Section 3. Section 4 discusses an example in which platform modularity and component sharing can be used simultaneously. Section 5 analyses the proposed model and discusses its effectiveness with a numerical illustration. Concluding remarks are made in Section 6, with a few suggestions for further extensions of the proposed model.

2. Literature review

There are many established studies that cover platform modularity and component sharing individually. Important and generalised results have been cited in those papers, some of which are used in this paper. The globalisation of industries in the recent past has resulted in increased research attention towards the need for product variety and the associated increase in complexity (Kerke and Srinivasan 1990, Fisher et al. 1994, Ramdas and Shawney 1998). Agard and Kusiak (2004) worked on and developed data-mining-based methodology to fill the need for the development of new techniques for designing product families.

Osborne and Armacost (1996) developed the quality aspect of product development, stating that an important ingredient in the product development process is the identification and subsequent optimisation of those product characteristics which denote quality. In an effort to meet the challenge of providing cost-effective variety, firms have begun viewing their related product offerings as a family, and share components, sub assemblies and production steps. These steps do not particularly encompass the product-planning phase of the development process but mainly are focused on the downstream component selection and process design tasks (Lee and Tang 1997, Sanderson and Uzumeri 1996, Gupta and Krishna 1999).

Meyer and his colleagues (Meyer and Utterback 1993, Meyer and Lehnard 1997) focused mainly on the benefits of the platform and illustrated the considerable significance of the platform modularity for developing product families. An exception is the work of Robertson and Ulrich (1998), who considered the potential loss of product differentiation because of platforms along with beneficial effects on design efficiency. Wang and Lin (2009) presented an overlapping process model to analyse the impact of process structure on the lead time of a complex development project. In addition to the literature reviewed thus far, the reader is also referred to the following more
recent literature: Chen and Wang (2008) used clustering analysis and Shannon’s entropy theory for the design of product platform, and Farrell and Simpson (2009) developed approaches on platform sharing and depicted the profitability of the design of product platforms.

There are certain shortcomings in the existing literature addressing similar problems, some of which have been tackled in this paper. Li and Azarm (2002) developed the approach for product line design selection and their sales depending on utility but did not differentiate between common platforms and components, which this paper considers. Michalek et al. (2006), discussed the effect of platforms on sales of the product family in the market, an important factor included in this research.

3. Model conceptualisation and formulation

We begin this section by describing the scenario for our hybrid model. A firm developing technology-based products manufactures two products. The first is a high end product with more features (product A) and consequently has a higher price compared to the other low end product with fewer features (product B). In response to this diversity, the firm can (a) introduce two separate products developed independently, (b) introduce two separate products but based on a common platform, (c) offer one product that serves both specifications, or (d) introduce only one specified product and not develop the other product. It has been proven that the introduction of two separate products based on a common platform is more profitable than the other scenarios in most conditions (Krishna and Gupta 2001). A platform is a set of subsystems and interfaces developed to form a common structure from which a stream of derivative products can be efficiently developed and produced. In this case, a platform is the common base shared across the whole product family on which the variants are developed. The remaining part of the variants can further have components that can be shared but are not part of the platform. This is termed as component sharing. Moreover, a platform is common to the whole product family, but there may be a set of components that can be common to the subset of the family. However, to keep things simple, only a two-product family has been considered. This leads to four possible cases: (i) platform modularity and component sharing integrated together to develop the two products, (ii) only platform modularity is used with the given set of components not shared, (iii) products are developed individually with the given set of components shared, or (iv) products are developed individually with the given set of components not shared. These cases will be referred to as A1, A2, A3 and A4 respectively from here on in this paper. We use a performance-based approach to analyse if platform modularity merged with component sharing is the most profitable scenario.

We assume that the consumers using a specific product (product A or product B) are homogeneous in the amount of utility they derive from the products. Consumer valuation is used to describe the utility derived by a consumer, following the established literature. Following the existing literature, the consumer valuation is considered a linear function of its performance level (Moorthy and Png 1992). Valuation of product A will be greater than that of product B as the performance of product A is greater than that of product B. Valuations of products A and B are denoted as $V_a$ and $V_b$ respectively.

In this model, we suppose that each of the two products is made of three sets of components. Of the three sets, the first one is the base and can be a common platform or developed individually. The second set consists of components that can be shared between the two products, and the third set, that defines the individual characteristics of the two products, is different for both the products. The performance levels of the three above-mentioned sets for the two products, product A and product B, are denoted by $p_{d1}$, $p_{d2}$, $p_{d3}$ and $p_{b1}$, $p_{b2}$, $p_{b3}$ for each product respectively. The cost of designing and developing a product of performance $p$ is given by the function $A(p^\beta)$ based on the industrial observation that fixed cost is directly proportional to the performance of the product. Although the function is general enough to capture different types of economies, we will present closed form results only for the case $\beta = 2$.

If the firm opts to develop the two products based on a common platform, the platform is designed in such a manner that it has high modularity, and if it delivers a performance level $p_p$, the fixed cost of designing and developing the platform is denoted by $Pp_p$. Based on earlier observations by Krishna and Gupta (2001) about the complexities and cost intensive nature of platforms, we assume $P > A$, $P$ and $A$ being the constants for the fixed cost functions of development of platform and non-platform products respectively.

The unit variable cost of a product is given by the function $cp^c$, where $c$ and $y$ are constants. The factor of economy of scale plays a leading role in reducing variable costs of production. Economy of scale refers to the decreased per unit cost as output increases. The initial investment of capital is diffused over an increasing number of units of output,
therefore the marginal cost of producing a good or service is less than the average total cost per unit. Thus logically, it seems that platform-based product development will reduce the cost of production significantly compared to producing the products individually. When multiple units of the same products are produced, which when separated into platform and non-platform components, economy of scale comes into play. The variable cost of a product is \( ac \) \((0 < a < 1)\), where \( a \) is the coefficient of economy of scale of the non-platform components that are shared. It is clear that the smaller the value of \( a \), the lower the cost of production, leading to increased profits.

The platform approach has several benefits that modify the variable cost of products. Since a platform produces a large volume of products, more effort is involved in the designing of a platform leading to a design which uses resources more efficiently. The economy of scale in platforms is denoted by \( k \) \((0 < k < 1)\), and is called integration benefits of the platform. As economy of scale is to non-platform based products, integration benefit is to platform products. It \( k \) is multiplied by the unit variable cost. Though at first it seems that platforms have a large number of benefits, there are certain drawbacks to them. Some of them include the under design of high-specification products and the over design of low-specification products. This may divert us from the ideal conditions that we are assuming, and thus force us to take these points into consideration. As the cases of over design and under design are supplementary to each other, only the case of the under design of the high-specification product is considered. The analysis of the over design of low-specification products proceeds in a similar manner. The under design coefficient is multiplied by the higher-specification products only and is denoted by \( u_1 \) \((u_1 = 1)\). So for the components developed in a platform, the performance value for both the products becomes \( u_1p_{a1} \). Hence the performance levels of the three components for the case of platform sharing only are \( u_1p_{a1}, p_{a2}, p_{a3} \) and \( u_1p_{a1}, p_{b2}, p_{b3} \) for product A and product B respectively.

Similar to the case of platform production, there are ill effects of component sharing too. Subsequently another under design coefficient is taken into consideration and is denoted by \( u_2 \) \((u_2 = 1)\). Consequently the performance levels of the three components for the case of component sharing only are \( p_{a1}, u_2p_{a2}, p_{a3} \) and \( u_1p_{a1}, u_2p_{a2}, p_{b3} \) for product A and product B respectively.

For the current purpose, we are considering that when the two products are developed individually, they provide the best performance suited for the particular segments they represent, and deviation from these performances will lead to sales loss. This loss would be a function of the deviation and, for the sake of simplicity, this loss is considered to be inversely proportional to the change shown graphically in Figure 1(a). Hence, if the performance of product A due to under design coefficients becomes \((u_1p_{a1} + u_2p_{a2} + p_{a3})\), the sales will drop and is given by

\[
n_{a1} = n_a(u_1p_{a1} + u_2p_{a2} + p_{a3})/(p_{a1} + p_{a2} + p_{a3})
\]

and similarly for the second product, it will be

\[
n_{b1} = n_b[u_2(p_{b1} + p_{b2} + p_{b3}) - (u_1p_{a1} + u_2p_{a2} + p_{a3})]/(p_{b1} + p_{b2} + p_{b3}).
\]

Following the above model, all the relevant terms are calculated for each of the production techniques and are shown in Table 1.

Firstly, we formulate the case where products are developed on a common platform with the given components shared. The factors of economy of scale and integration benefit are taken into account for the calculation of variable

![Figure 1. Estimated sales value versus performance of products.](image)
costs. The element of economy of scale ($\alpha$) is multiplied by the variable cost of production of the shared set of components, whereas the integration benefit factor is multiplied by the variable cost of production of the set of components developed on the common platform. So the variable cost of production is given in Equation (1).

$$VarCost_1 = n_{a1}c(k(u_{1a1})\gamma + \alpha(u_{2a2})\gamma + (p_{a3})\gamma) + n_{b1}c(k(u_{1b1})\gamma + \alpha(u_{2b2})\gamma + (p_{b3})\gamma)$$

(1)

Fixed costs, on the other hand, consist of the costs of design and development of the sets of components. Hence fixed costs are given by Equation (2).

$$FxdCost_1 = A\left(2(u_{2a2})^\beta + p_{a3}^\beta + p_{b3}^\beta\right) + P(u_{1a1})^\beta$$

(2)

These were the expenses and now the revenue generated by the sales of the products is calculated. It is a function of consumer valuation, performance and expected sales. So the revenue function is given in Equation (3).

$$Revenue_1 = n_{a1}V_aQ_{a1} + n_{b1}V_bQ_{b1}$$

(3)

where

- $Q_{a1} = u_{1a1} + u_{2a2} + p_{a3}$
- $Q_{b1} = u_{1b1} + u_{2b2} + p_{b3}$
- $V_a$ Consumer valuation of product A
- $V_b$ Consumer valuation of product B
- $n_{a1}$ Estimated sale of product A for production technique 1
- $n_{b1}$ Estimated sale of product B for production technique 1

The variable cost of case 2 when only platform production is considered is similar to the first case. The factor of economy of scale ($\alpha$) is omitted as the set of components is developed individually. The rest of the equation will be formulated accordingly and is depicted in Equation (4).

$$VarCost_2 = n_{a2}c(k(u_{1a2})\gamma + (p_{a2})\gamma + (p_{a3})\gamma) + n_{b2}c(k(u_{1b2})\gamma + (p_{b2})\gamma + (p_{b3})\gamma)$$

(4)

Fixed costs will constitute costs of design and development of platform as well as the other two sets of components. The fixed cost equation is given by Equation (5).

$$FixedCost_2 = A\left(p_{a2}^\beta + p_{a3}^\beta + p_{b2}^\beta + p_{b3}^\beta\right) + P(u_{1a2})^\beta$$

(5)

Consequently the revenue equation becomes

$$Revenue_2 = n_{a2}V_aQ_{a2} + n_{b2}V_bQ_{b2}$$

(6)

where

- $Q_{a2} = u_{1a2} + u_{2a2} + p_{a3}$
- $Q_{b2} = u_{1b2} + u_{2b2} + p_{b3}$

Table 1. Relevant terms for each production technique.
The third situation is when only one set of components is shared and the other two sets are developed discretely. So only the economy of scale is considered and integration benefit is not incorporated.

\[
VarCost_3 = n_{a2}c((p_{a1})^y + \alpha(p_{a2})^y + (p_{a3})^y) + n_{b2}c((p_{a1})^y + \alpha(p_{a2})^y + (p_{b3})^y)
\]  

(7)

Fixed costs will comprise only the costs of design and development of the two sets of components developed individually and the shared component. The fixed cost of the platform is not to be considered. Thus, the fixed cost equation turns out to be

\[
FxdCost_3 = A\left(2(u_2p_{a2})^\beta + p_{a2}^\beta + p_{b3}^\beta + p_{a1}^\beta + p_{b1}^\beta\right)
\]  

(8)

The revenue function is given by Equation (9).

\[
A((u_2p_{a2})^y + p_{a3}^y + p_{b3}^y + p_{a1}^y + p_{b1}^y)
\]  

(9)

In the final case, we neither consider the platform modularity nor the component sharing and so, it is the most abridged case amongst the four. The variable cost function is thus given by

\[
VarCost_4 = n_{a4}c((p_{a1})^y + (p_{a2})^y + (p_{a3})^y) + n_{b4}c((p_{a1})^y + (p_{a2})^y + (p_{b3})^y)
\]  

(10)

The fixed cost consists of the design and development costs of all the six sets of components. The function turns into:

\[
FixedCost_4 = A\left(p_{a2}^\beta + p_{b2}^\beta + p_{a3}^\beta + p_{b3}^\beta + p_{a1}^\beta + p_{b1}^\beta\right)
\]  

(11)

Revenue function becomes:

\[
Rev_4 = n_{a4}V_aQ_{a4} + n_{b4}V_bQ_{b4}
\]  

(12)

4. Illustrative example

The model developed in the previous section is now demonstrated by considering an actual product. Though there are various examples available in the production industry having three sets of components as the performance criteria, we dedicate our research work to the automobile industry, taking a four-wheel automobile as our main illustration. We take the instance of two cars of the same model differing in the type of engine incorporated. According to the model proposed in this paper, product A is a petrol car and product B, a diesel car. The engine and various special features that may differentiate depending upon the customers’ needs are included in the third set of components that defines the individual entity of the product. In the case of common-platform-based production, the chassis of the car will be the common platform and hence the looks of the two models will be similar. And if the firm does not choose the option of developing the two products on a common platform, the two cars may have different appearances. Had the firm opted for common-platform-based production, the company would have saved the designing cost of the aerodynamic structure of the car but then the company would have had to design the platform in such a manner that both the models could be developed on the same platform. Also the factor of the under design of high-end products or the over design of low-end products comes into play. Depending upon the dominance of the two factors, the firm should make a decision that results in maximum profitability.

Keeping the above-mentioned points in mind as well as the demand of the customers’ specifications, the firm should decide judiciously which method to adopt. There are certain components in an automobile which can be shared between the two products, for example the battery, the steering, the internal features and so on. Consider the case of tires for instance. As the petrol engine is more powerful than a diesel engine, better quality tires should be used for the petrol car. The reason being that a petrol car has more torque, hence better quality tires are needed for the efficiency of braking and the safety of passengers. But the firm may opt to use the same tires for both cars. The factor of economy of scale comes into the picture which results in savings of variable costs. Again the customers’ demand specifications are to be taken into consideration by the firm.
5. Model analysis

5.1 Model

In this section, we will analyse the four approaches for profitability and the conditions that must be applied for a particular approach to be most lucrative. The equations formulated in the previous section are generic and although the model is general enough to tackle different types of economies, we set the values of $\beta$ and $\gamma$ as 2, as stated earlier in this paper. This will also help us to get close form results and we will be able to compare the profits of different approaches. The profit functions for all the approaches are given by Equations (13)–(16). Gross profit is the difference between the revenue and the total cost of production which includes fixed and variable costs.

\[
\text{Profit}_1 = n_{a1}V_aQ_{a1} + n_{b1}V_bQ_{b1} - \left[ n_{a1}c(k(u_1p_{a1})^2 + \alpha(u_2p_{a2})^2 + (p_{a3})^2) + n_{b1}c(k(u_1p_{a1})^2 + \alpha(u_2p_{a2})^2 + (p_{b3})^2) + A(2(u_2p_{a2})^2 + p_{a3}^2 + p_{b3}^2) + P(u_1p_{a1})^2 \right] 
\]

\[
\text{Profit}_2 = n_{a2}V_aQ_{a2} + n_{b2}V_bQ_{b2} - \left[ n_{a2}c(k(u_1p_{a1})^2 + (p_{a2})^2 + (p_{a3})^2) + n_{b2}c(k(u_1p_{a1})^2 + (p_{b2})^2 + (p_{b3})^2) + A(p_{a2}^2 + p_{a3}^2 + p_{b2}^2 + p_{b3}^2) + P(u_1p_{a1})^2 \right] 
\]

\[
\text{Profit}_3 = n_{a3}V_aQ_{a3} + n_{b3}V_bQ_{b3} - \left[ n_{a3}c((p_{a2})^2 + \alpha(u_2p_{a2})^2 + (p_{a3})^2) + n_{b3}c((p_{a2})^2 + \alpha(u_2p_{a2})^2 + (p_{b2})^2 + (p_{b3})^2) + A(2(u_2p_{a2})^2 + p_{a3}^2 + p_{b2}^2 + p_{b3}^2) + P(u_1p_{a1})^2 \right] 
\]

\[
\text{Profit}_4 = n_{a4}V_aQ_{a4} + n_{b4}V_bQ_{b4} - \left[ n_{a4}c((p_{a2})^2 + (p_{a2})^2 + (p_{a3})^2) + n_{b4}c((p_{a2})^2 + (p_{a2})^2 + (p_{b3})^2 + (p_{b3})^2) + A(p_{a2}^2 + p_{a3}^2 + p_{b2}^2 + p_{b3}^2 + p_{b3}^2) + P(u_1p_{a1})^2 \right] 
\]

The above model consists of a number of parameters and so, instead of opting for a direct mathematical comparison based on the following four profitability equations, we will consider a descriptive example to compare the results and use graphical analysis methods to arrive at imperative conclusions.

5.2 Case study

In this model, we have four variable parameters, specifically $k$, $\alpha$, $u_1$ and $u_2$. Our aim is to locate the perfect balance between the four variables (if they are included) which results in maximum profitability. Inspired by two real products, a realistic numerical data set is developed. Based on this data set, we will first analyse the pattern in profitability varying all four parameters individually. This will depict the behavioural trend of each profit function with the variation of these parameters. Another salient feature of this model is to obtain an estimate of the profits for the respective approaches given a particular set of values. Numerous sets of values were analysed to compare different approaches and the following set was chosen so that the maximum number of cases could be demonstrated.

\[
\begin{align*}
  p_{a1} &= 70 & p_{a2} &= 40 & p_{a3} &= 50 \\
  p_{b1} &= 50 & p_{b2} &= 30 & p_{b3} &= 40 \\
  u_1 &= 0.9 & u_2 &= 0.9 & k &= 0.86 \\
  n_a &= 1000 & n_b &= 1500 & \alpha &= 0.86 \\
  c &= 0.3 & A &= 140 & P &= 260 \\
  V_a &= 27.5 & V_b &= 26.5
\end{align*}
\]

Considering this data set, the profit of each of the approaches (A1, A2, A3 and A4) is evaluated. But the purpose of this section is to depict the changes in profit with the individual variation of different parameters.

Figure 2 illustrates the effect of varying the under design coefficient in common platforms on the various approaches. The minimum value of the under design coefficient will be when $u_1p_{a1} = p_{b1}$, and the maximum value will be when $u_1p_{a1} = p_{a1}$. Hence the range of $u_1$ is from 0.7 to 1. Approaches A3 and A4 are independent of the coefficient of under design in the platforms and subsequently they do not change with the change in $u_1$. It is clear from the graph in Figure 2 that as we increase $u_1$, both A1 and A2 decrease considerably. Referring to Figure 2, it is evident that the company will benefit the most when the value of $u_1$ is 0.7 and approach A2 is adopted.
interpretation of this is that only the lower specification product’s platform may be used as common for the benefit of the firm. But the firm has to decide judiciously, keeping in mind consumer demand as well. If it is not possible for the designers to design the higher specification product based on the lower specification product’s platform, any value between 0.7 and 0.87 may be incorporated and the result will still be lucrative for approach A2. But if due to any constraints, the value of $u_1$ lies above 0.87, the firm should opt for approach A3, which is component sharing only.

Figure 3 is the plot between the under design coefficient for component sharing and profit. As discussed in the previous case, the value of $u_2$ will range from $p_b$ to $p_a$, that is to say between 0.75 and 1. In this scenario A2 and A4 are independent of $u_2$ and thus the horizontal line is justified. This case reveals close competition between A1, A2 and A3. All of the approaches are profitable for certain values of $u_2$. However this graph is for the certain set of values which we have taken and may vary depending on the firm’s data. So the approach that should be adopted entirely depends on the value of $u_2$ that the firm can afford.

The dependence of profit on the value of $k$ for all the approaches is depicted in Figure 4. We assume the value of $k$ ranges from 0.5 to 1 as practically it cannot be lower than 0.5. Although the profit decreases as we increase the value of $k$, A1 and A2 are still the best approaches for the firm till the value of $k$ becomes 0.86. If we further increase the value of $k$, A3 becomes most profitable, even though it is independent of $k$. The difference of this case from the first case is that the firm cannot decide on the value of $k$. They have to decide the approach by making an estimate of the value of $k$, if the common platform is built, and then decide if A1 or A2 will be profitable. If the firm is unable to decrease the integration benefit to 0.86, only component sharing shall be favoured. It may be noted that the graph in Figure 4 is dependent on the data set considered. Consequently it depends on the value of $\alpha$ too, if the actual value of $\alpha$ is greater than the assumed value, A2 shall be adopted, and if it is less, A1 will be more lucrative for the firm.

Figure 5 shows a graphical representation between $\alpha$ and profit. Similar to the case of $k$, $\alpha$ also varies from 0.5 to 1. $\alpha$ is the coefficient of the scale economy for component sharing and therefore it will only affect A1 and A3. A2 and A4 will be indifferent towards $\alpha$. The graph in Figure 5 clearly expresses which technique will be most profitable for the firm.

An in-depth research of the variation of the values are given in Figures 6 and 7. These are the plots between $\alpha$ and $u_1$ and $u_2$, respectively. We first describe the plot between $\alpha$ and $k$ in Figure 6. It depicts the approaches that shall be adopted by the firm for the given value of $\alpha$ (Y axis) and $k$ (X axis). For a particular set of values of $\alpha$ and $k$, different approaches may be profitable. These are shown in the areas given in the graph. So, the appropriate approach may be selected once the firm knows the value of the integration benefit of a platform and the economy of scale of component sharing.

Similarly, Figure 7, which is the graph between $u_1$ and $u_2$, gives an idea about the specification of the product to be developed for different approaches to be most profitable. If the firm has predetermined the specification of the product, then by using the values of $u_1$ and $u_2$, the most lucrative approach can be known. These graphs will be of
great use for the firm, as they will clearly give an idea about which approach to implement. We can see from all the graphs that approach A1 is most likely to be more profitable than the other approaches.

Figure 8(a–c) are the graphs in which the difference between various profit functions are plotted on the $Z$ axis, $k$ is varied along the $X$ axis and $\frac{1}{C_{11}}$ along the $Y$ axis. The plots show the variance of the advantage of one approach over the other when economies of scale and integration benefit are varied along the $x$ and $y$ axis respectively. In Figure 8(a), we see that A1 is most profitable over A2 at $k = 1, \alpha = 0.5$. But in Figure 8(b), at the same point ($k = 1, \alpha = 0.5$), we see that A3 is more profitable than A1. Hence, we need to find the combinations of $\alpha$ and $k$ for which each process is profitable. The values of $k$ and $\alpha$ above the $z = 0$ line represents that the first function, $P_1$ in Figure 8, is more profitable. Hence, we merge all the three plots that are given in Figure 8(a–c) and the plane is cut along $z = 0$. The two-dimensional plot obtained is given in Figure 6. In Figure 6, we can easily find the values for which each approach is more lucrative.

We now use $u_1$ and $u_2$ to compare the four approaches. In Figure 9(a–c), the $Z$ axis shows the variance of the differences between the profits of two approaches while $u_1$ and $u_2$ are varied along the $X$ and $Y$ axis, respectively. As done for $\alpha$ and $k$, a similar approach is applied for these three plots and the two-dimensional plot obtained between $u_1$ and $u_2$ is shown in Figure 7. From Figure 7, we can now easily predict the most profitable approach for the different combinations of $u_1$ and $u_2$.

Suppose the values of $\alpha$ and $k$ are available to the firm, they can directly locate from the plot in Figure 6 which approach should they opt for in order to maximise their returns. Otherwise, if the firm is planning to design a whole
new product family, using the graphs of profit versus individual parameters in Figures 2–5, the range of each parameter can be estimated for the optimal design of a product family using the hybrid model.

6. Conclusion

The model developed was applied on a realistic numerical data set and the behaviour of various parameters was analysed. Although due to computing limitations, the model could not be solved analytically, an example was considered and the findings were as follows:

(1) When the two approaches (component sharing and platform modularity) are combined, it does result in a more profitable product family for a considerable range of parameters compared to the other approaches applied individually or not used at all.

(2) Based on the data set considered, the range of values of various parameters (k, α, u1 and u2) for the most profitable product family was obtained.

(3) For certain scenarios, A1 was not the most lucrative approach. In such cases, the optimal approach among A2, A3 and A4 could be found.

Based on the findings, this model can be applied to:

(1) Develop a whole new product family. For this method, the firm has to make an estimate or find the data set for their product family. Having already shown that A1 is the most beneficial approach for a wide range of parametric values, the firm can easily obtain the range of values of the concerned parameters by making a similar model.

(2) Select optimal product development approach. To know, with the existing parameter settings, which approach the firm should opt for in order for the results to be the most beneficial.
As the example was not taken from a particular industry, it is apparent that the model can be used for product development in almost all sorts of industry.

In the present study, the product family of two products is derived. De Weck et al. (2003) and Bhandare and Allada (2009) developed methodologies to obtain the optimal number of platforms required for a given product family. In future, this can be extrapolated to a bigger product family. Also, all the components to be shared are considered in a single set but a new study considering multiple sets for more closely related components is possible. The linear effect of platforms in the market can be replaced by more complex and appropriate functions like the Taguchi loss function, among others.

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